

Probing the Perturbative NLO Parton Evolution in the Small- x Region

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Abstract

A dedicated test of the perturbative QCD NLO parton evolution in the very small- x region is performed. We find a good agreement with recent precision HERA-data for $F_2^p(x, Q^2)$, as well as with the present determination of the curvature of F_2^p . Characteristically, perturbative QCD evolutions result in a positive curvature which increases as x decreases. Future precision measurements in the very small x -region, $x < 10^{-4}$, could provide a sensitive test of the range of validity of perturbative QCD.

Parton distributions $f(x, Q^2)$, $f = q, \bar{q}, g$, underlie a Q^2 -evolution dictated by perturbative QCD at $Q^2 \gtrsim 1 \text{ GeV}^2$. It was recently stated [1] that the NLO perturbative QCD Q^2 -evolution disagrees with HERA data [2, 3] on $F_2^p(x, Q^2)$ in the small- x region, $x \lesssim 10^{-3}$. In view of the importance of this statement we perform here an independent study of this issue. In contrast to [1] we shall undertake this analysis in the standard framework where one sets up input distributions at some low Q_0^2 , here taken to be $Q_0^2 = 1.5 \text{ GeV}^2$, corresponding to the lowest Q^2 considered in [1], and adapting these distributions to the data considered. In the present case the data considered will be restricted to

$$1.5 \text{ GeV}^2 \leq Q^2 \leq 12 \text{ GeV}^2, \quad 3 \times 10^{-5} \lesssim x \lesssim 3 \times 10^{-3} \quad (1)$$

as in [1] and will be taken from the corresponding measured $F_2^p(x, Q^2)$ of the H1 collaboration [2]. The choice of these data is motivated by their higher precision as compared to corresponding data of the ZEUS collaboration [3], in particular in the very small- x region.

We shall choose two sets of input distributions based on the GRV98 parton distributions [4]. In the first set we shall adopt $u_v, d_v, s = \bar{s}$ and $\Delta \equiv \bar{d} - \bar{u}$ from GRV98 and modify $\bar{u} + \bar{d}$ and the gluon distribution in the small- x region to obtain an optimal fit to the H1 data [2] in the aforementioned kinematical region. We shall refer to this fit as the ‘best fit’. The second choice will be constrained to modify the GRV98 $\bar{u} + \bar{d}$ and g distributions in the small- x region as little as possible. We shall refer to this fit as GRV_{mod}. It will turn out that both input distributions are compatible with the data to practically the same extent, i.e. yielding comparable χ^2/dof . In view of these observations we do not agree with the conclusions of ref. [1], i.e. we do not confirm a disagreement between the NLO Q^2 -evolution of $f(x, Q^2)$ and the measured [2, 3] Q^2 -dependence of $F_2^p(x, Q^2)$.

The remaining flavor-singlet input distributions at $Q_0^2 = 1.5 \text{ GeV}^2$ to be adapted to

the recent small- x data are expressed as

$$xg(x, Q_0^2) = N_g x^{-a_g} (1 + A_g \sqrt{x} + 7.283x) (1 - x)^{4.759} \quad (2)$$

$$x(\bar{u} + \bar{d})(x, Q_0^2) = N_s a^{-a_s} (1 + A_s \sqrt{x} - 4.046x) (1 - x)^{4.225} \quad (3)$$

where the parameters relevant for the large x -region, $x > 10^{-3}$, which is of no relevance for the present small- x studies, are kept unchanged and are taken from, e.g. GRV98 [4]. The refitted relevant small- x parameters turn out to be

$$\begin{aligned} \text{'best fit'} : \quad N_g &= 1.70, \quad a_g = 0.027, \quad A_g = -1.034 \\ N_s &= 0.171, \quad a_s = 0.177, \quad A_s = 2.613 \end{aligned} \quad (4)$$

$$\begin{aligned} \text{GRV}_{\text{mod}} : \quad N_g &= 1.443, \quad a_g = 0.125, \quad A_g = -2.656 \\ N_s &= 0.270, \quad a_s = 0.117, \quad A_s = 1.70 \end{aligned} \quad (5)$$

to be compared with the original GRV98 parameters [4]: $N_g = 1.443$, $a_g = 0.147$, $A_g = -2.656$ and $N_s = 0.273$, $a_s = 0.121$, $A_s = 1.80$. The resulting predictions are compared to the H1-data [2] in Fig. 1. These results are also consistent with the ZEUS-data [3] with partly lower statistics. The corresponding χ^2/dof are 0.50 for the ‘best fit’ ($dof = 48$) and 0.94 for GRV_{mod} ($dof = 50$), respectively. Our treatment of the heavy flavor contributions to F_2 differs from that in [1]. We evaluate these contributions in the fixed flavor $f = 3$ scheme of [4], together with the massive heavy quark (c, b) contributions, rather than in the $f = 4$ (massless) scheme utilized in [1]. We have checked, however, that our disagreement with [1] does not result from our $f = 3$ plus heavy quarks vs. the $f = 4$ massless quark calculations in [1]: we have also performed a fit for $f = 4$ massless quarks and the results for F_2 and its curvature, to be discussed below, remain essentially unchanged.

In Figs. 2 and 3 we show our gluon and sea input distributions in (2) and (3), as well as their evolved shapes at $Q^2 = 4.5 \text{ GeV}^2$ in the small- x region. It can be seen that both

of our new small- x gluon distributions at $Q^2 = 4.5 \text{ GeV}^2$ conform to the rising shape obtained in most available analyses published so far, in contrast to the valence-like shape obtained in [1] where the gluon density xg decreases as $x \rightarrow 0$. It is possible to conceive a valence-like gluon at some very-low Q^2 scale, as in [4], but even in this extreme case the gluon ends up as non valence-like at $Q^2 > 1 \text{ GeV}^2$, in particular at $Q^2 = 4.5 \text{ GeV}^2$, as physically expected.

Turning now to the curvature test of F_2 advocated and discussed in [1], we first present in Fig. 4 our results for $F_2(x, Q^2)$ at $x = 10^{-4}$, together with two representative expectations of global fits [5, 6], as a function of [1]

$$q = \log_{10} \left(1 + \frac{Q^2}{0.5 \text{ GeV}^2} \right). \quad (6)$$

This variable has the advantage that most measurements lie along a straight line [1] as indicated by the dotted line at $x = 10^{-4}$ in Fig. 4. The MRST01 parametrization [5] results in a sizable curvature for F_2 in contrast to all other fits shown in Fig. 4. This large curvature, incompatible with the data presented in [1], is mainly caused by the valence-like input gluon distribution of MRST01 at $Q_0^2 = 1 \text{ GeV}^2$ in the small- x region which becomes even negative for $x < 10^{-3}$ [5]. A similar result was obtained in [1] based on a particular gluon distribution $xg(x, Q^2)$ which decreases with decreasing x for $x \lesssim 10^{-3}$ even at $Q^2 = 4.5 \text{ GeV}^2$ (cf. fig. 7 in [1]). More explicitly the curvature can be directly extracted from

$$F_2(x, Q^2) = a_0(x) + a_1(x)q + a_2(x)q^2. \quad (7)$$

The curvature $a_2(x) = \frac{1}{2} \partial_q^2 F_2(x, Q^2)$ is evaluated by fitting the predictions for $F_2(x, Q^2)$ at fixed values of x to a (kinematically) given interval of q . In Fig. 5(a) we present $a_2(x)$ which results from experimentally selected q -intervals [1]:

$$\begin{aligned} 0.7 \leq q \leq 1.4 & \quad \text{for} \quad 2 \times 10^{-4} < x < 10^{-2} \\ 0.7 \leq q \leq 1.2 & \quad \text{for} \quad 5 \times 10^{-5} < x \leq 2 \times 10^{-4} \\ 0.6 \leq q \leq 0.8 & \quad \text{for} \quad x = 5 \times 10^{-5}. \end{aligned} \quad (8)$$

Notice that the average value of q decreases with decreasing x due to the kinematically more restricted Q^2 range accessible experimentally. For comparison we also show in Fig. 5(b) the curvature $a_2(x)$ for an x -independent fixed q -interval

$$0.6 \leq q \leq 1.4 \quad (1.5 \text{ GeV}^2 \leq Q^2 \leq 12 \text{ GeV}^2). \quad (9)$$

Apart from the rather large values of $a_2(x)$ specific for the MRST01 fit as discussed above (cf. fig. 4), our ‘best fit’ and GRV_{mod} results, based on the inputs in (4) and (5), respectively, do agree well with the experimental curvatures as calculated and presented in [1] using H1 data. It should be noted that perturbative NLO evolutions result in a positive curvature $a_2(x)$ which increases as x decreases. This feature is supported by the data shown in fig. 5(a); since the data point at $x < 10^{-4}$ is statistically insignificant, future precision measurements in this very small x -region should provide a sensitive test of the range of validity of perturbative QCD evolutions.

Furthermore, the H1 collaboration [2] has found a good agreement between the perturbative NLO evolution and the slope of $F_2(x, Q^2)$, i.e. the first derivative $\partial_{Q^2} F_2$.

To conclude, the perturbative NLO evolution of parton distributions in the small- x region is compatible with recent high-statistics measurements of the Q^2 -dependence of $F_2^p(x, Q^2)$ in that region. A characteristic feature of perturbative QCD evolutions is a positive curvature $a_2(x)$ which increases as x decreases (cf. fig. 5). Although present data are indicative for such a behavior, they are statistically insignificant for $x < 10^{-4}$. Future precision measurements and the ensuing improvements of the determination of the curvature in the very small x -region should provide further information concerning the detailed shapes of the gluon and sea distributions, and perhaps may even provide a sensitive test of the range of validity of perturbative QCD.

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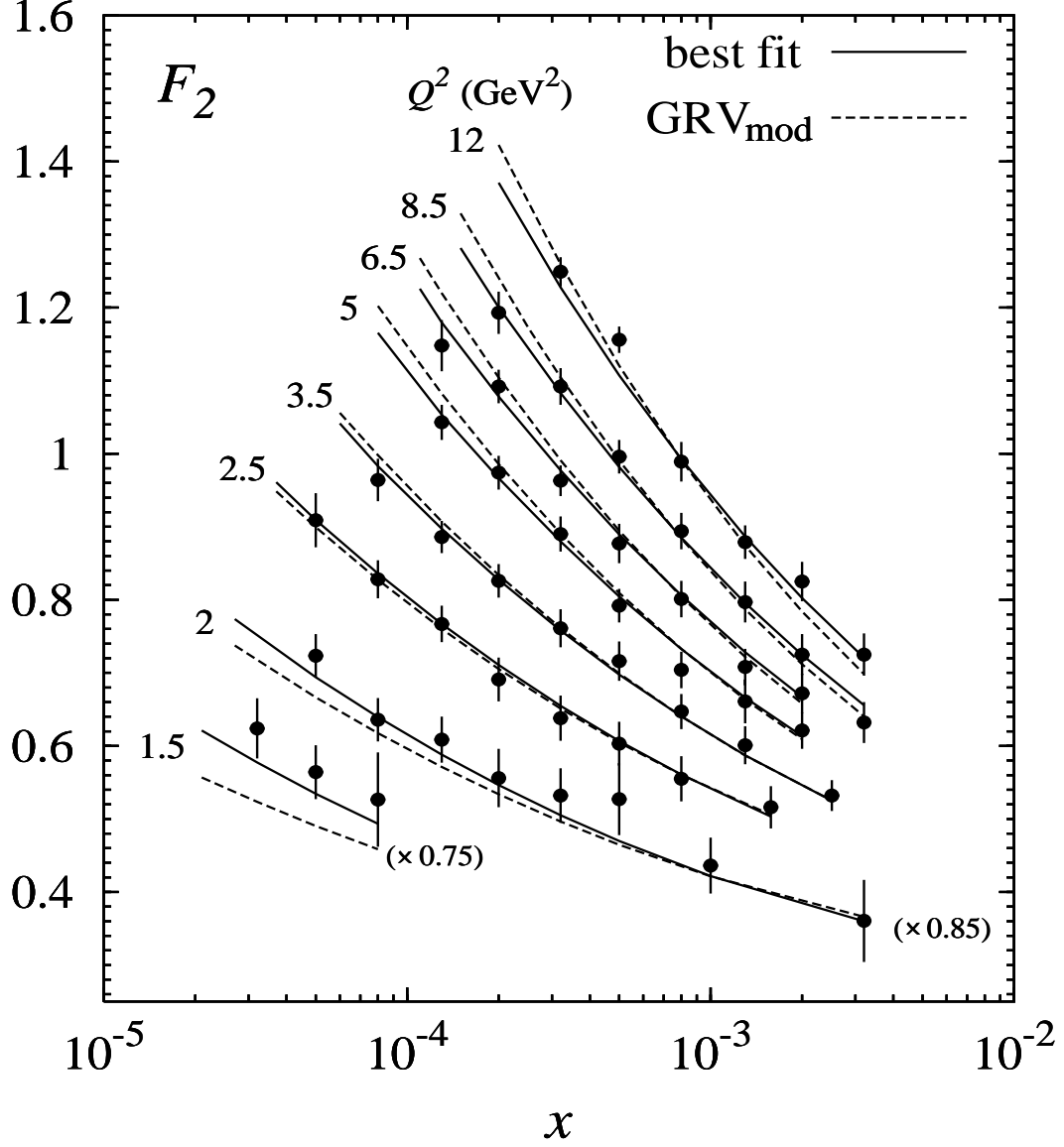


Figure 1: Comparison of our ‘best fit’ and GRV_{mod} results based on (4) and (5), respectively, with the H1 data [2]. To ease the graphical representation, the results and data for the lowest bins in $Q^2 = 1.5 \text{ GeV}^2$ and 2 GeV^2 have been multiplied by 0.75 and 0.85, respectively, as indicated

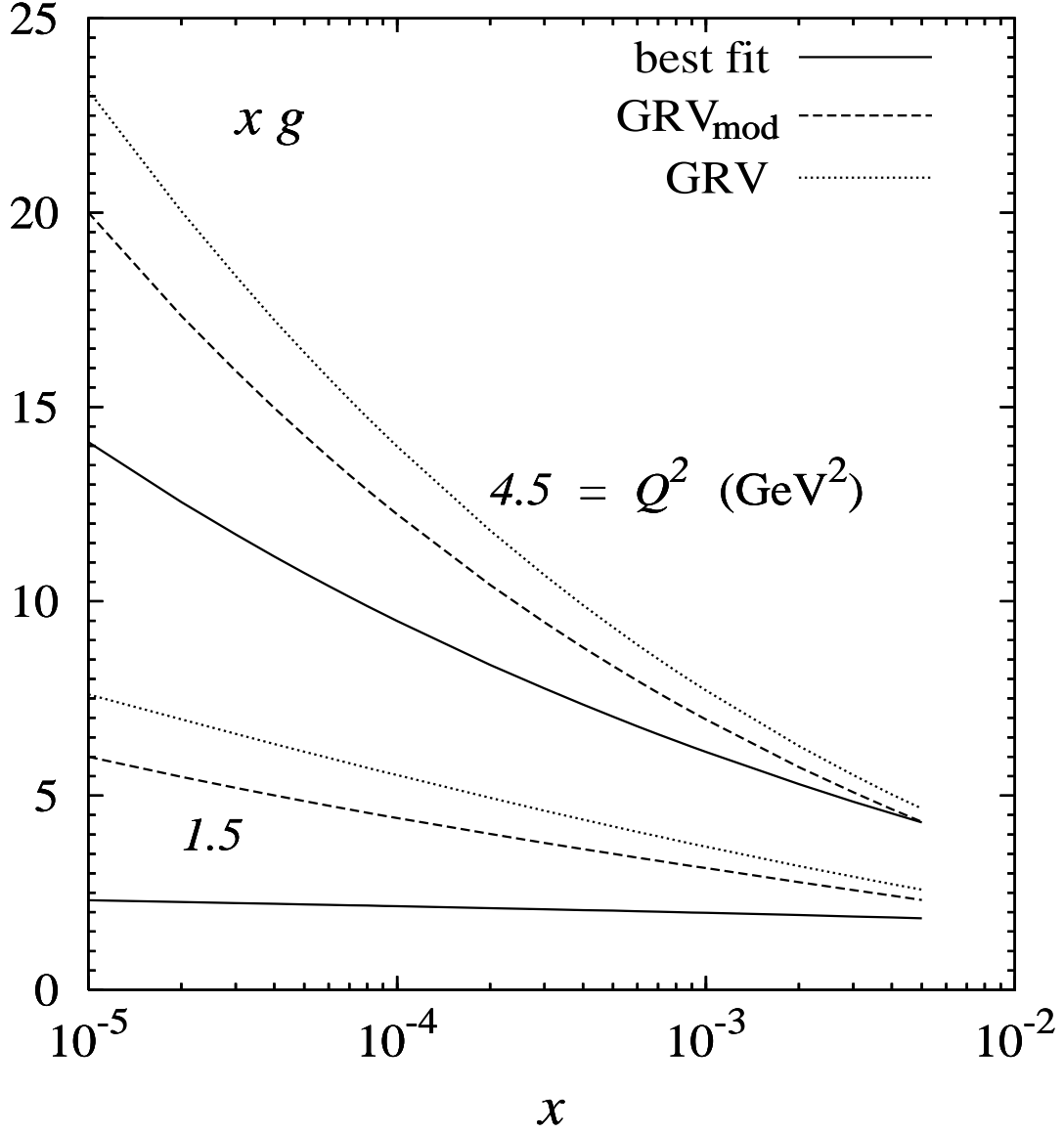


Figure 2: The gluon distributions at the input scale $Q_0^2 = 1.5 \text{ GeV}^2$ corresponding to (2) with the ‘best fit’ and GRV_{mod} parameters in (4) and (5), respectively, and at $Q^2 = 4.5 \text{ GeV}^2$. For comparison, the original GRV98 results [4] are shown as well by the dotted curves

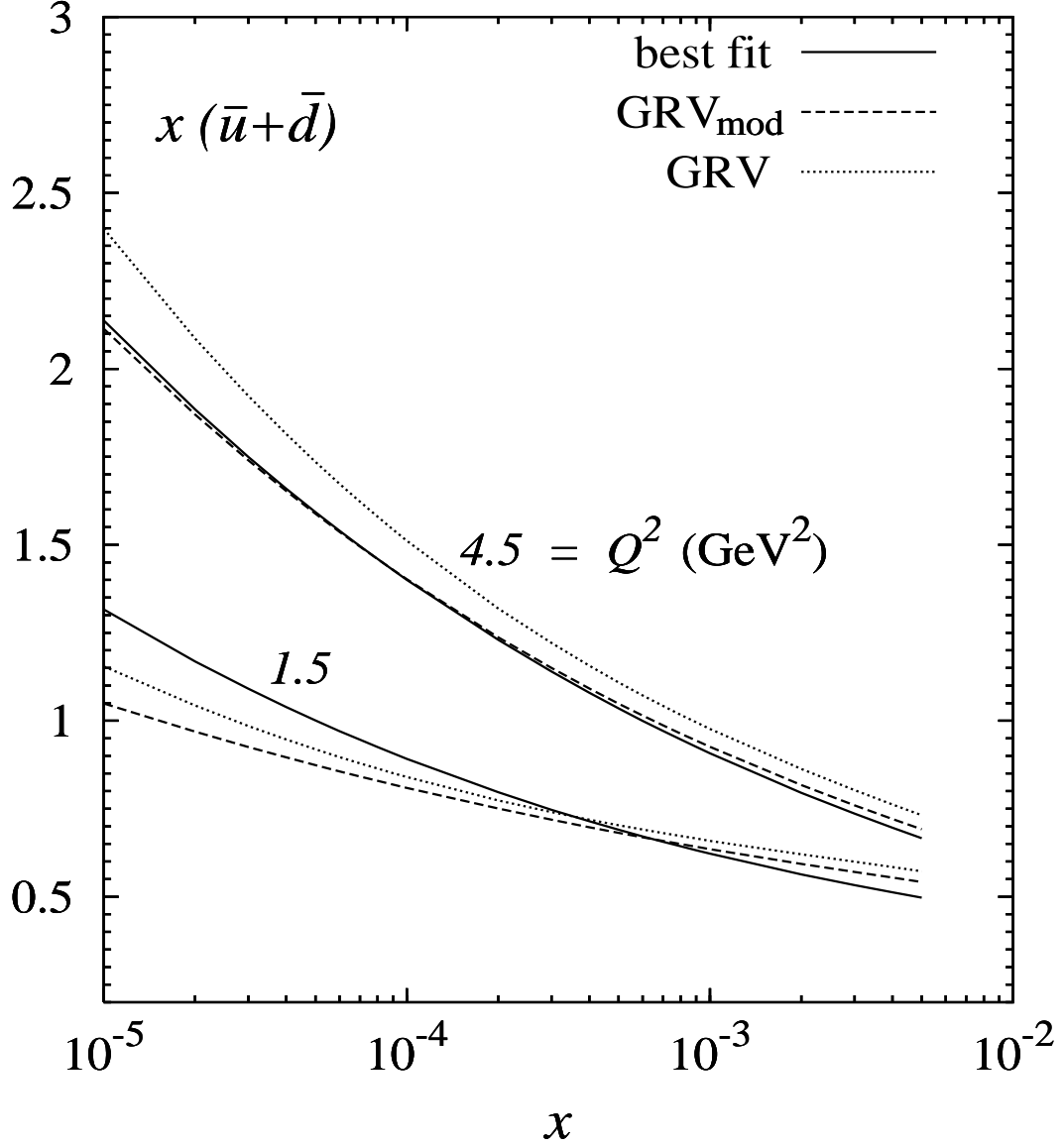


Figure 3: The sea distribution $x(\bar{u} + \bar{d})$ at the input scale $Q_0^2 = 1.5 \text{ GeV}^2$ in (3) with the ‘best fit’ and GRV_{mod} parameters in (4) and (5), respectively, and at $Q^2 = 4.5 \text{ GeV}^2$. For comparison, the original GRV98 results [4] are shown as well by the dotted curves

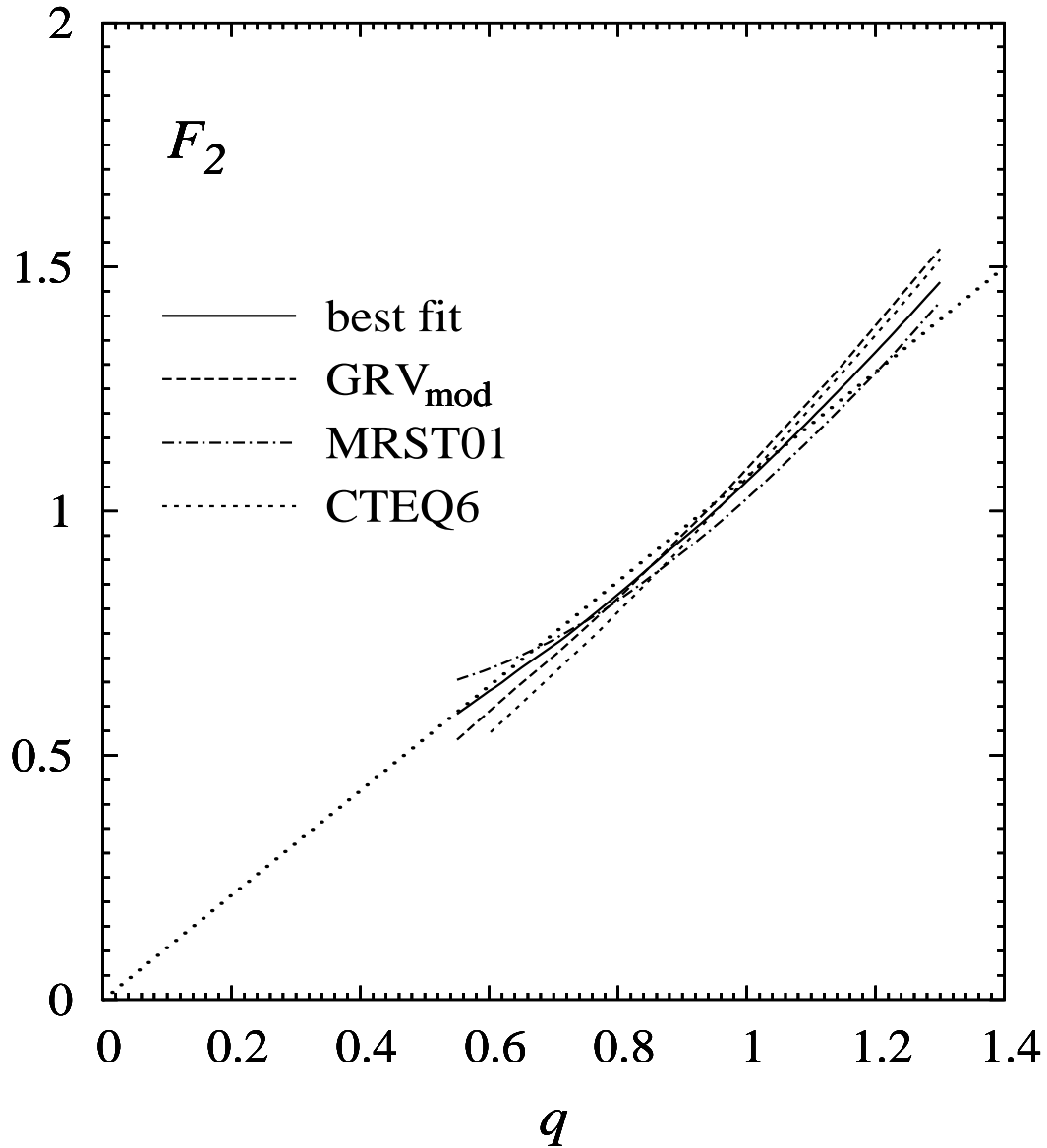


Figure 4: Predictions for $F_2(x, Q^2)$ at $x = 10^{-4}$ plotted versus q defined in (6). Representative global fit results are taken from MRST01 [5] and CTEQ6M [6]. Most small- x measurements lie along the straight (dotted) line [1]

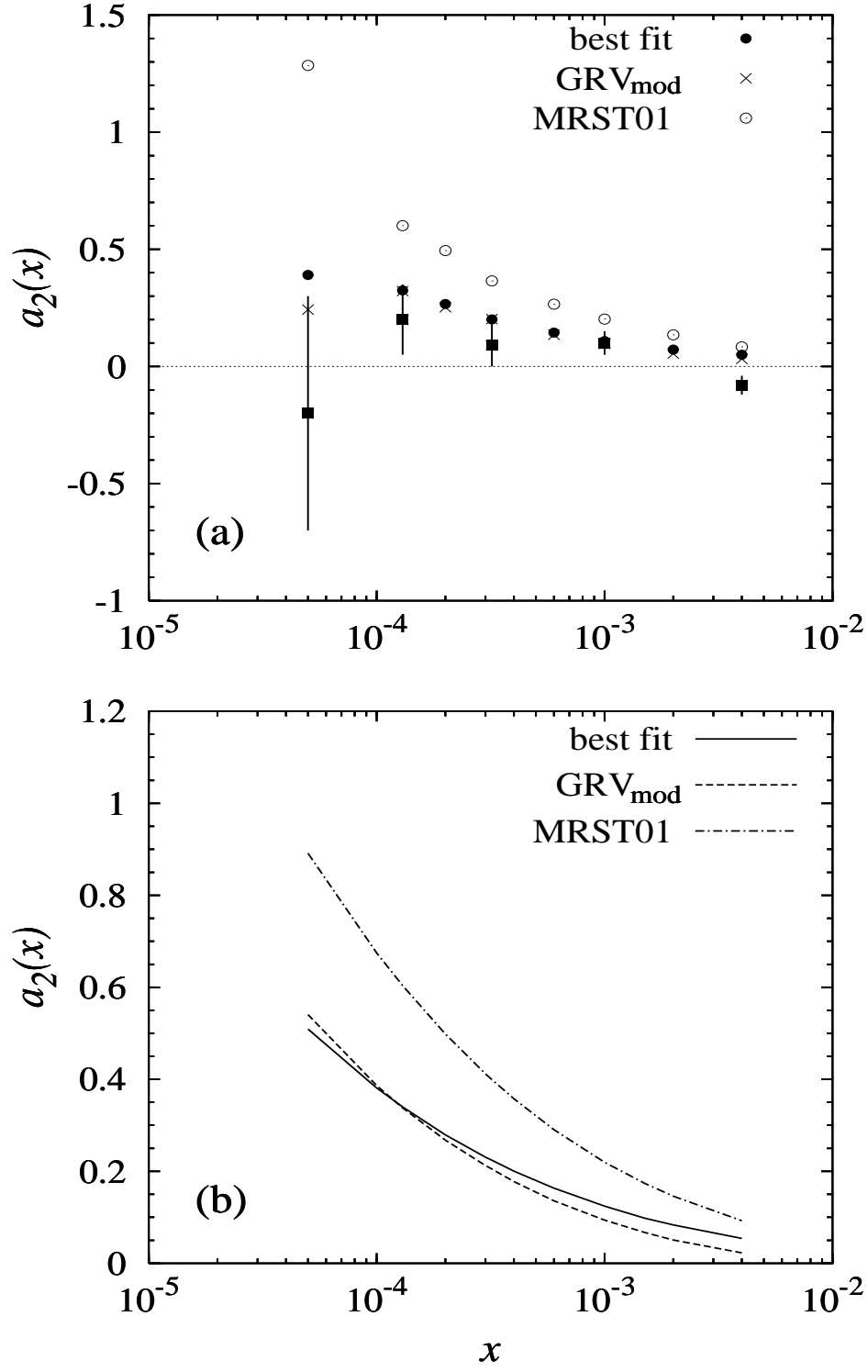


Figure 5: The curvature $a_2(x)$ as defined in (7) for (a) the variable q -intervals in (8) and (b) the fixed q -interval in (9). Also shown are the corresponding MRST01 results [5]. The experimental curvatures (squares) shown in (a) are taken from [1]